

## HARDY INEQUALITY IN THE GENERALIZED LEBESGUE SPACES

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*Dedicated to Professor Gary Roach,  
on the occasion of his 70th birthday*

### Abstract

The Hardy type inequality

$$\left\| |x - x_0|^{\beta - \alpha} \int_{\Omega} \frac{f(y) dy}{|y - x_0|^{\beta} |x - y|^{n - \alpha}} \right\|_{L^{p(\cdot)}(\Omega)} \leq C \|f\|_{L^{p(\cdot)}(\Omega)}, \quad 0 < \alpha < n,$$

$x_0 \in \overline{\Omega}$ , is proved for the spaces  $L^{p(\cdot)}(\Omega)$  with variable exponent  $p(x)$  in the case of bounded domains  $\Omega$  in  $R^n$ ,  $-\frac{n}{p(x_0)} < \beta < \frac{n}{q(x_0)}$ .

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